from a small doubt about the first-order term, this advantage is likely to be maintained in more general circumstances.

3. $<u_2^2>$ Measured with an X Wire

Similar considerations for this case lead to

$$<u_2^2>_{\text{meas}} = <u_2^2>_{\text{act}}[1 - 2(<u_2^2u_3^2>/U^2<u_2^2>) + \ldots]$$

and so indicate an 8% error for linearized operation in the particular circumstances just specified. Although there is still likely to be an improvement in most circumstances over, say, constant-current operation, note that this error is of the order-of-magnitude more usually associated with nonlinearized systems.

4. Reynolds Stress Measured with an X Wire

This case is of particular interest, because the first-order terms are of the same form as the turbulent-transport terms in the energy-balance equation of a turbulent shear flow, and existing experimental information may be used to assess them. For linearized operation

$$<u_1u_2>_{\text{meas}} = <u_1u_2>_{\text{act}}[1 + (<u_2u_3^2>/U<u_1u_2>) - 2(<u_1u_2u_3^2>/U^2<u_1u_2>) + \ldots]$$

If, as in two-dimensional flow, $<u_1\, u_2\, u_3^2> = <u_1\, u_2>\cdot<u_3^2>$, the second-order term gives 8% error with 20% intensity. Experimental data on turbulent shear flows suggest that a typical value of $|<u_2u_3^2>/<u_1u_2><u_1^2>^{1/2}|$ is 0.5, and, in this case, the first-order term gives 10% error for 20% intensity. (The two terms may be of the same or opposite sign according to the circumstances.) This suggests that, for the greater intensities at which these points are most important, second-order terms probably will be more serious than first-order terms. By way of comparison, constant-current operation gives

$$\begin{aligned}<u_1u_2>_{\text{meas}} = <u_1u_2>_{\text{act}}\Bigg[1 &- \frac{3}{4}(1+2\alpha)\frac{<u_1^2u_2>}{U<u_1u_2>} - \frac{1}{4}(1+2\alpha)\frac{<u_2^3>}{U<u_1u_2>} + \frac{<u_2u_3^2>}{U<u_1u_2>} + \\ &\frac{5+12\alpha+12\alpha^2}{8}\left(\frac{<u_1^3u_2> + <u_1u_2^3>}{U^2<u_1u_2>}\right) - \\ &\frac{7+6\alpha}{2}\frac{<u_1u_2u_3^2>}{U^2<u_1u_2>} - \frac{1+12\alpha+20\alpha^2}{8}\times \\ &\left(\frac{<u_1^2> + <u_2^2>}{U^2}\right) + \frac{1+6\alpha}{2}\frac{<u_3^2>}{U^2} + \ldots\Bigg]\end{aligned}$$

where α is the same as in Eq. (2-45) of Ref. 2. The first-order terms usually will give larger errors than in linearized operation. The second-order terms are too complicated for any general assessment, but for the particular case of $<u_1^2> = <u_2^2> = <u_3^2>$, $<u_1u_2u_3^2> = <u_1u_2>\cdot<u_3^2>$, and $<u_1^3\, u_2> = <u_1\, u_2^3> = (8/\pi)^{1/2}<u_1u_2><u_1^2>$ (chosen because $<|u^3|> = (8/\pi)^{1/2}<u^2>^{3/2}$ for a Gaussian distribution), these terms give a markedly smaller error than the terms in a linearized operation. This is a very restricted inference, but in general there is perhaps little to choose between the two modes of operation for $<u_1u_2>$ measurements.

As already implied, these considerations are illustrative, not definitive. But the authors think that they lead to the following (interrelated) inferences: that the advantages of linearization are sufficiently marginal that it may not always be worth the trouble and expense; that the weight given to measurements with linearized equipment as opposed to nonlinearized ought not to be so much greater; and that the accuracy of results from linearized equipment needs to be assessed in terms of the particular quantity and circumstances. In connection with this last point, it may be considered an advantage of linearization that the estimation of errors is relatively straightforward, not merely because the algebra is less laborious but also because it involves fewer quantities whose magnitude has to be guessed.

Of course, the pros and cons of different methods cannot be considered in terms of accuracy alone. Some people express a preference for linearized operation because it is more straightforward to use. However, one gets the impression that measurements with linearized equipment sometimes have been given greater weight, although accuracy may not have been the reason for selecting this technique. The main point is that this calls for caution.

Since preparation of this note, a paper by Rose[4] has appeared dealing with the same matter, though from a different point of view.

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Deceleration and Its Higher Time Derivatives for Objects During Atmospheric Entry

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Expressions are derived for the first and second time derivatives of deceleration of an object during atmospheric entry. The altitudes at which the maximum and minimum values of these derivatives occur are determined. The differences between these altitudes and the altitude at which maximum deceleration occurs are shown to be constants that depend only on the atmospheric density. The maximum and minimum values of the first and second time derivatives of deceleration are determined. It is shown that these values are independent of the drag characteristics of the object.

Nomenclature

V = velocity of entering object along trajectory (positive in direction of motion)
t = time
C_D = drag coefficient
ρ_0 = atmospheric density at earth's surface
A = reference area for drag evaluation
V_E = initial velocity of entering object
β = atmospheric density coefficient such that $\rho = \rho_0 e^{-\beta y}$
m = mass of object
θ_E = angle of entry (measured with respect to local horizon)

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y = altitude of object
y_{max} = altitude at which peak deceleration occurs
$(dV/dt)_{y_{max}}$ = peak deceleration

In a paper by Allen and Eggers,[1] expressions for the velocity and deceleration of an object during atmospheric entry are derived assuming a constant drag coefficient and an exponential variation of atmospheric density with altitude. Expressions for the peak deceleration encountered during entry and the altitude at which this peak value occurs also are determined.

When the effects of gravity are neglected, these expressions are as follows [Eqs. (13–15, and 17) of the Allen and Eggers paper]:

$$V = V_E \exp\left(-\frac{C_D\rho_0 A}{2\beta m \sin\theta_E}\right) e^{-\beta y} \tag{1}$$

$$\frac{dV}{dt} = -\frac{C_D\rho_0 A V_E^2}{2m} e^{-\beta y} \exp\left(-\frac{C_D\rho_0 A}{\beta m \sin\theta_E}\right) e^{-\beta y} \tag{2}$$

$$y_{max} = \frac{1}{\beta}\log_e\left(\frac{C_D\rho_0 A}{\beta m \sin\theta_E}\right) \tag{3}$$

$$\left(\frac{dV}{dt}\right)_{y_{max}} = -0.18394\ \beta V_E^2 \sin\theta_E \tag{4}$$

By differentiation of Eq. (2), one obtains

$$\frac{d^2V}{dt^2} = -\left(\frac{C_D\rho_0 A}{2m}\right)(V_E^3)(e^{-\beta y})\left[\exp\left(-\frac{3C_D\rho_0 A}{2\beta m \sin\theta_E}\right)\times e^{-\beta y}\right]\left[\beta\sin\theta_E - \left(\frac{C_D\rho_0 A}{m}\right)e^{-\beta y}\right] \tag{5}$$

To obtain the second time derivative, Eq. (5) in turn may be differentiated with respect to time. This yields

$$\frac{d^3V}{dt^3} = -\frac{C_D\rho_0 A\ V_E^4}{2m}\left[\exp\left(-\frac{2C_D\rho_0 A}{\beta m \sin\theta_E}\right)e^{-\beta y}\right]\times (Qe^{-\beta y} - Re^{-2\beta y} + Se^{-3\beta y}) \tag{6}$$

where

$$Q = \beta^2\sin^2\theta_E \qquad R = \frac{7\beta C_D\rho_0 A\sin\theta_E}{2m}$$

$$S = \frac{3C_D^2\rho_0^2 A^2}{2m^2}$$

The altitudes at which the maximum and minimum values of the first time derivative of deceleration occur can be determined by differentiating Eq. (5) with respect to time and setting the expression obtained after differentiation [Eq. (6)] equal to zero. This gives a quadratic equation for y which, when solved and combined with Eq. (3), yields the altitudes at which the maximum and minimum rate of change of deceleration occurs:

$$y_{max\,1,1} = \frac{1}{\beta}\log_e\left(\frac{3C_D\rho_0 A}{\beta m\sin\theta_E}\right) = y_{max} + \frac{1.1}{\beta} \tag{7}$$

and

$$y_{max\,1,2} = \frac{1}{\beta}\log_e\left(\frac{C_D\rho_0 A}{2\beta m\sin\theta_E}\right) = y_{max} - \frac{0.69}{\beta} \tag{8}$$

(Here, as in the rest of this paper, the designation $y_{max\ j,k}$ is used, where y_{max} indicates an altitude at which a maximum or minimum occurs, j indicates the order of the derivative of the deceleration, and k denotes the number of the maximum or minimum value in order of descending altitude.)

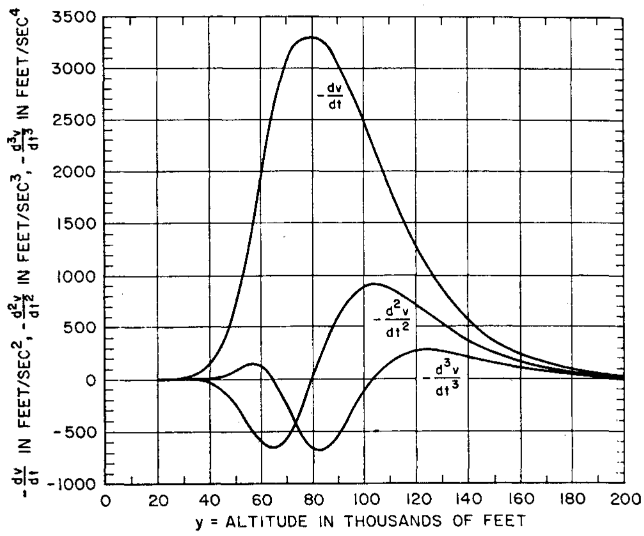

Fig. 1 Deceleration and its two higher time derivatives (V_E = 25,000 fps, θ_E = 40°, W/C_DA = 100 psf)

As β is assumed a known constant, Eqs. (7) and (8) indicate that the vertical distances between the altitude of maximum deceleration and the altitudes of maximum and minimum rate of change of deceleration are constants. The peak values of the rate of change of deceleration are obtained by substituting Eqs. (7) and (8) into Eq. (5). This yields

$$d^2V/dt^2_{y_{max\,1,1}} = -0.06739\beta^2 V_E^3\sin^2\theta_E \tag{9}$$

and

$$d^2V/dt^2_{y_{max\,1,2}} = 0.04979\beta^2 V_E^3\sin^2\theta_E \tag{10}$$

Note that these peak values are independent of the drag characteristics of the entry object.

To determine the maximum and minimum values of the second time derivative of the deceleration, Eq. (6) may be differentiated with respect to y and set equal to zero. This

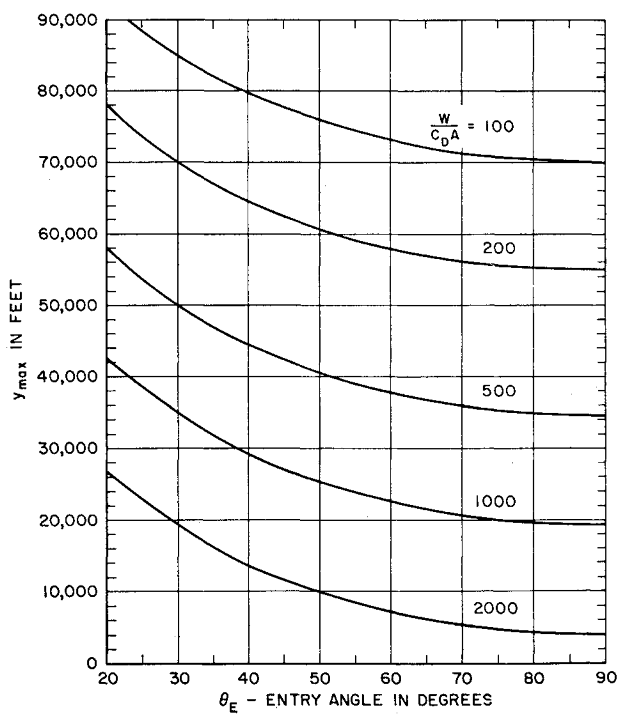

Fig. 2 Altitude at which maximum deceleration occurs

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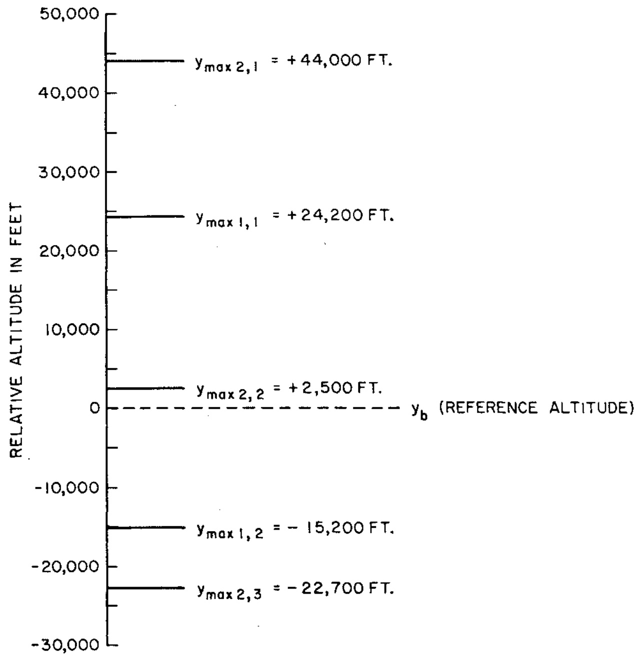

Fig. 3 Relative altitudes at which the peak values of time derivatives occur ($\beta = 1/22{,}000$ ft^{-1})

gives a cubic equation, which, when solved and combined with Eq. (3), gives

$$y_{\max 2,1} = \frac{1}{\beta}\log_e \frac{7.52\, C_D\rho_0 A}{\beta m \sin\theta_E} = y_{\max} + \frac{2.02}{\beta} \tag{11}$$

$$y_{\max 2,2} = \frac{1}{\beta}\log_e \frac{1.12\, C_D\rho_0 A}{\beta m \sin\theta_E} = y_{\max} + \frac{0.113}{\beta} \tag{12}$$

$$y_{\max 2,3} = \frac{1}{\beta}\log_e \frac{C_D\rho_0 A}{2.81\beta m \sin\theta_E} = y_{\max} - \frac{1.03}{\beta} \tag{13}$$

Again it is seen that, for a constant β, the vertical distances between the altitude of maximum deceleration and the altitudes of maximum and minimum values for the second derivative of deceleration are constants. To determine the maximum and minimum values of d^3V/dt^3, substitute the values for $y_{\max}$ in Eqs. (11–13) back in Eq. (6). This yields

$$d^3V/dt^3_{y_{\max 2,1}} = -0.0286\, \beta^3 V_E{}^4 \sin^3\theta_E \tag{14}$$

$$d^3V/dt^3_{y_{\max 2,2}} = 0.070\beta^3 V_E{}^4 \sin^3\theta_E \tag{15}$$

$$d^3V/dt^3_{y_{\max 2,3}} = -0.0153\beta^3 V_E{}^4 \sin^3\theta_E \tag{16}$$

Again, it is seen that these values are independent of the drag characteristics of the object.

Graphic Results

Figure 1 is a plot of the deceleration and its two higher time derivatives as a function of altitude for a typical object entering the atmosphere with the following parameters: V_E = 25,000 fps, θ_E = 40°, W/C_DA = 100 psf, and β = 1/22,000 ft^{-1}. Deceleration has been taken as equal to $-dV/dt$ and thus appears as a positive quantity. In a similar fashion, the time rate of change of deceleration equals $-d^2V/dt^2$, and the second time derivative of deceleration equals $-d^3V/dt^3$.

The other curves facilitate determination of the peak magnitudes of deceleration and its time derivatives and the altitudes at which these peaks occur. Figure 2 gives the altitudes of peak deceleration as a function of entry angle and object ballistic coefficient, W/C_DA. As shown in Eq. (3), this altitude is independent of entry velocity.

Figure 3 gives the relative altitudes of the peaks of the higher time derivatives referenced to the altitude of peak deceleration assuming that β = 1/22,000 ft^{-1}. From Figs. 2 and 3 the altitude at which any maximum or minimum of the two higher time derivatives of deceleration occurs may be determined.

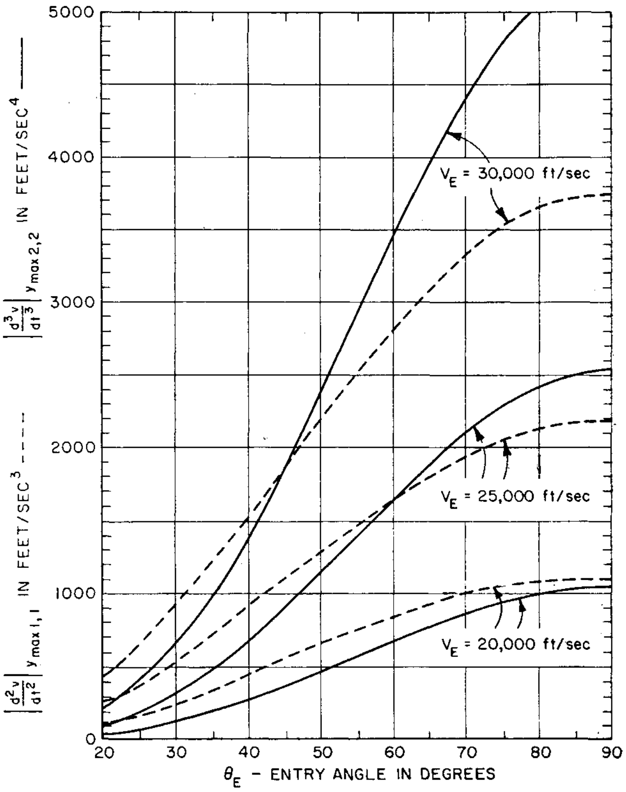

Fig. 4 Maximum magnitudes of time derivatives

Figure 4 gives the maximum magnitudes reached for the two higher time derivatives of deceleration as a function of entry angle and velocity. These values are independent of the drag characteristic of the object.

Thermal Deflection of a Circular Sandwich Plate

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A differential equation is derived for the thermal deflection of a circular sandwich plate. The effect of core shear rigidity is included in the analysis. A particular solution is obtained for a simply supported plate with uniform and unequal temperatures of the two face plates. In addition, expressions are presented for the radial and circumferential bending stresses. It is shown that the plate center deflection is dependent on core rigidity only for values of the rigidity parameter $\bar{X}$ less than 10. Beyond this value the deflection is essentially the same as with infinite core shear rigidity.

SOME assumptions of this analysis are 1) axial symmetry ($\partial/\partial_\theta = 0$), 2) $\sigma_r = 0$ and $\sigma_\theta = 0$ in the core, 3) core is assumed isotropic with shear rigidity G_c psi, and 4) transverse

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